NAME: ______________________________

MATH 8 SAMPLE TEST 3

No scratch paper No graphing calculators.

This test is in two parts. On part one, you may not use a calculator; on part two, a calculator is necessary. When you complete part one, tear it off and place it at the front of your desk, I will collect it. Once you have turned in part one, you may not go back to it.

PART ONE - NO CALCULATORS ALLOWED

(1) Find each of the following: (Note: here, answers to inverse trig. problems should be in radians, not degrees) (2 points each)

(a) $\cos^{-1}\left(\frac{-\sqrt{2}}{2}\right) =$ $\frac{3\pi}{4}$

(b) $\cos^{-1}(0)$ $\frac{\pi}{2}$

(c) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) =$ $\frac{\pi}{6}$

(d) $\cos(315^\circ) =$ $\frac{\sqrt{2}}{2}$

(e) $\sin(120^\circ) =$ $\frac{\sqrt{3}}{2}$

(f) $\sin^{-1}(2) =$ undefined

(g) cos(0)= 1

(h) $\sin^{-1}(-1) =$ $-\frac{\pi}{2}$

(i) $\cot\left(\frac{7\pi}{4}\right) =$ -1

(j) $\sin^{-1}\left(-\frac{1}{2}\right) =$ $-\frac{\pi}{6}$

(k) $\sin^{-1}\left(\underbrace{\sin\left(\frac{5\pi}{3}\right)}_{-\frac{\sqrt{3}}{2}}\right) =$ $-\frac{\pi}{3}$

(l) $\tan(135^\circ) =$ -1

(2) HOW MANY solutions does each of the following equations with the given restrictions on θ have? (Do not need to solve, just tell how many solutions there would be.) (1 point each)

(a) $\sin\theta = \frac{1}{5};\ 0 \le \theta < 2\pi$ 2

(c) $\tan\theta = -7;\quad 0 \le \theta \le \pi$ 1

(b) $\theta = \sin^{-1}(0.3)$ 1

(d) $\cos\theta = \frac{2}{7}$ unfinitely many

(3) Solve the following equations exactly. (all solutions) (3 points each)

(a) $\cos^2\theta - 1 = 0$

$\cos^2\theta = 1$

$\cos\theta = \pm 1$

$\theta = \pi k$, k integer

(b) $\sin\left(\frac{x}{3}\right) = \frac{\sqrt{2}}{2}$

$\frac{x}{3} = \frac{\pi}{4} + 2\pi k,\ \frac{3\pi}{4} + 2\pi k$

$x = \frac{3\pi}{4} + 6\pi k,\ \frac{9\pi}{4} + 6\pi k$

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(4) Solve the following equations exactly for $0 \le \theta \le 2\pi$. ___________ (3 points each)

(a) $\tan(2x) = \sqrt{3}$

$2x = \frac{\pi}{3} + \pi k$

$x = \frac{\pi}{6} + \frac{\pi}{2}k$

$x = \frac{\pi}{6} + \frac{3\pi}{6}k$

$x = \frac{\pi}{6},\ \frac{2\pi}{6},\ \frac{7\pi}{6}\ \frac{5\pi}{3}$

(b) $4\cos\theta - 2 = 0$

$4\cos\theta = 2$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3},\ \frac{5\pi}{3}$

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(5) Solve the following equations exactly for $0 \le \theta \le 2\pi$. (3 points each)

(a) $\cos\theta = \frac{1}{4}$

$\theta = \cos^{-1}\frac{1}{4},\ 2\pi - \cos^{-1}\frac{1}{4}$

(b) $\sin\theta = -0.3$

ref $= \sin^{-1}(0.3)$

$\theta = \pi + \sin^{-1}(0.3),$

$2\pi - \sin^{-1}(0.3)$

(c) $\tan\theta = 5$

$\theta = \tan^{-1}5,$

$\pi + \tan^{-1}5$

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MATH 8 Sample Test 3
PART TWO - CALCULATORS ALLOWED (no graphing calc.)
Show your work on this paper. EXACT answers are expected unless otherwise specified.
Fill in the blanks with the most appropriate, simplified answer.

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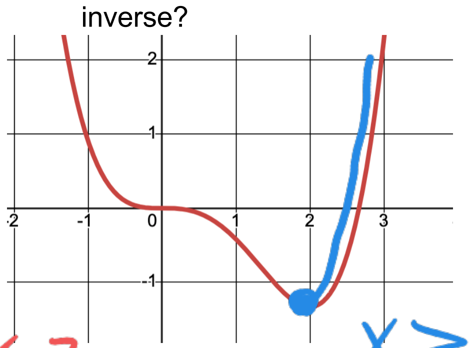
(6) The graph of a function is given. What restriction would you make so that the restricted function has an inverse? (2 points)



$x \le 2$ OR $x \ge 2$

we learned how to do this 3 diff. ways. Any is fine

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(7) Given that $\tan(\theta) = -\frac{2}{3}$ and θ is in Quadrant III, find the values of the other 5 trig functions of θ exactly (show work) (10 points)

① Using identities

$\tan^2\theta + 1 = \sec^2\theta$

$\left(-\frac{2}{3}\right)^2 + 1 = \sec^2\theta$

$\frac{4}{9} + 1 = \sec^2\theta$

$\frac{13}{9} = \sec^2\theta$

$\sec\theta = \pm\sqrt{\frac{13}{9}} \Rightarrow \frac{\sqrt{13}}{3}$ Q3

$\Rightarrow \cos\theta = \frac{3}{\sqrt{13}}$.

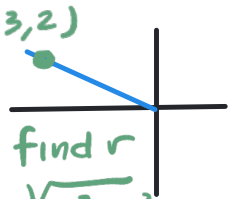
② Using defintion when given point (x,y) on terminal side

$\tan = \frac{-2}{3} = \frac{y}{x}$

but in Q3 so $x<0$, $y>0$

$\Rightarrow y = 2$, $x = -3$

(-3,2)



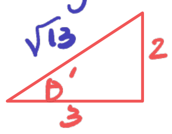
find r

$r = \sqrt{x^2+y^2}$
$= \sqrt{4+9} = \sqrt{13}$

$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{13}}$ etc (*)

③ Using right △ def.

let θ' be the acute angle with $\tan\theta' = \frac{+2}{3} = \frac{opp}{adj}$



Find Hyp using Pythag. Thm.
$\Rightarrow \sqrt{13}$

$\cos\theta' = \frac{adj}{hyp} = \frac{3}{\sqrt{13}}$

In Q3, cosine < 0 so $\cos\theta = -\frac{3}{\sqrt{13}}$

⋮

etc

$\sin(\theta) =$ $-2/\sqrt{13}$ $\cos(\theta) =$ $-3/\sqrt{13}$ $\sec(\theta) =$ $-\sqrt{13}/3$

$\csc(\theta) =$ $-\sqrt{13}/2$ $\cot(\theta) =$ $-3/2$

$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \sin\theta = \tan\theta\cos\theta$
$= \left(\frac{-2}{3}\right)\left(\frac{3}{\sqrt{13}}\right) = -2/\sqrt{13}$ etc

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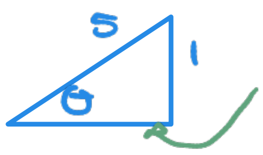
(8) Evaluate exactly: $\cos\left(\sin^{-1}\left(\frac{1}{5}\right)\right)$ (You must show work, calculator may not be used). (3 points)

With any inverse trig.. that we cant simplify, it can be helpful to call it θ

So let $\theta = \sin^{-1}\left(\frac{1}{5}\right)$

$\Rightarrow \sin\theta = 1/5$ AND $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

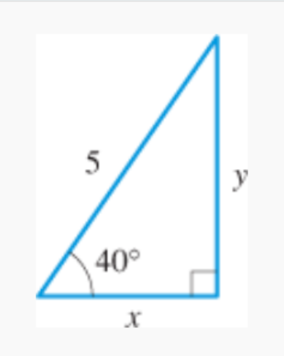


$1^2 + x^2 = 25$
$x^2 = 24$
$x = \sqrt{24}$

$\cos\left(\sin^{-1}\frac{1}{5}\right)$

$= \cos\theta = \frac{\sqrt{24}}{5} = \frac{2\sqrt{6}}{5}$

(9) Solve for x and y, exactly (4 points)



$\cos 40° = \frac{ADJ}{HYP} \Rightarrow \cos 40° = \frac{x}{5} \Rightarrow x = 5\cos 40°$

$\sin 40° = \frac{OPP}{HYP} \Rightarrow \sin 40° = \frac{y}{5} \Rightarrow y = 5\sin 40°$

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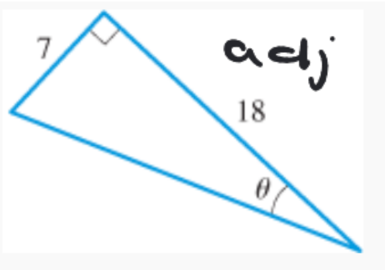
(10). Solve for θ exactly: (2 points)

opp



$\tan\theta = \frac{opp}{adj} = \frac{7}{18}$

$\theta = \tan^{-1}\left(\frac{7}{18}\right)$

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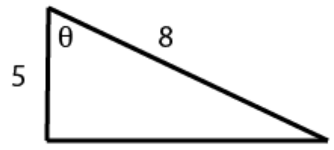
(11) This problem checks your understanding one three versions of the definitions of the trigonometric functions. Given the following figures, find: (2 points each)

pythag...

$\sqrt{64-25} = \sqrt{39}$



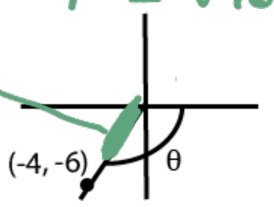
$\sqrt{39}$

Find r

$r^2 = (-4)^2 + (-6)^2$

$r = \sqrt{16+36} = \sqrt{52}$

$= 2\sqrt{13}$

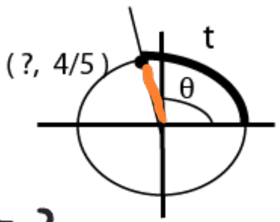


$\cos\theta = \frac{x}{r} = \frac{-4}{2\sqrt{13}} = \frac{-2}{\sqrt{13}}$

$x^2 + y^2 = 1$

$x^2 + \frac{16}{25} = 1 \nearrow x^2 = 1 - \frac{16}{25}$

$x^2 = \frac{9}{25}$



$x = -\frac{3}{5}$

$\tan\theta = \frac{y}{x} = \frac{4/5}{-3/5}$

$= -\frac{4}{3}$

(a) $\tan\theta =$ $\sqrt{39}/5$ (c) $\cos\theta =$ $-2/\sqrt{13}$ (e) $\tan t =$ $-4/3$

approx

(b) $\theta \approx$ ______ degrees (d) $\theta \approx$ ______ degrees (f) $t \approx$ ______ radian mode

(t is a *number, not angle*)

$\theta = \tan^{-1}\left(\frac{\sqrt{39}}{5}\right)$ $\theta = \cos^{-1}\left(-\frac{2}{\sqrt{13}}\right)$ $t = \pi - \text{ref} = \pi - \tan^{-1}\left(\frac{4}{3}\right)$

positive

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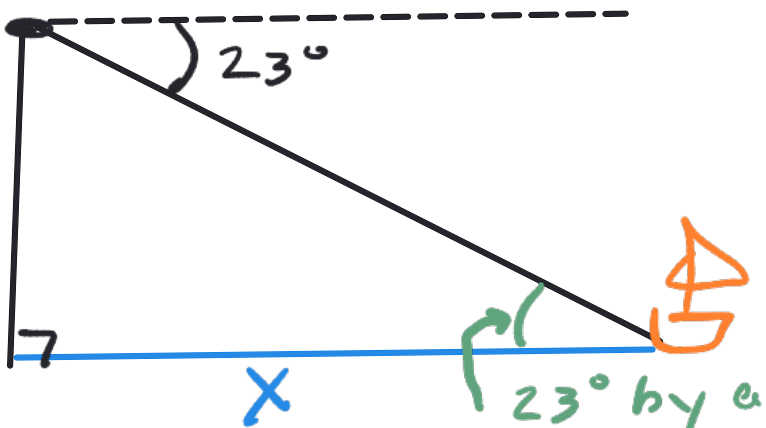
(12) An person sitting at the top of a 300 foot cliff at the edge of the ocean observes a ship directly offshore. The angle of depression from the person to the ship is 23 degrees. How far is the ship from shore (exact and approximate) (4 points

opp 23°

300 ft



adj 23°

23° by alternate interior angles

$\tan 23° = \frac{300}{x}$

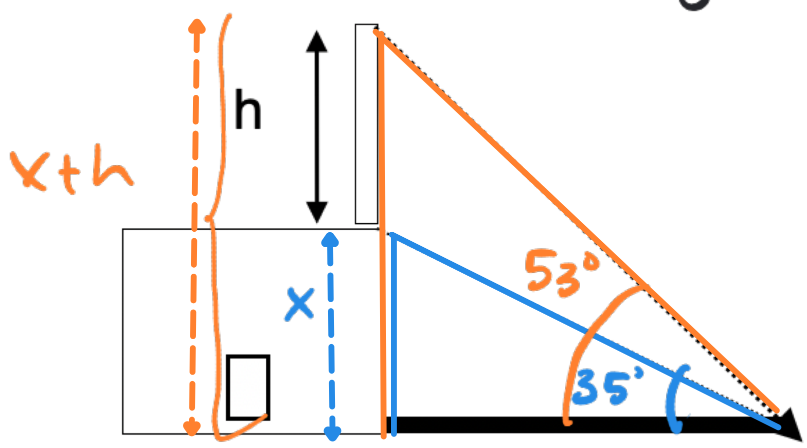
$x\tan 23° = 300$

$x = \frac{300}{\tan 23°} \approx$

Note: You might have done it diffently but approx. should be same

(13) At a point on the ground 200 feet from the base of a building, the angle of elevation to the bottom of a smokestack on the top of the building is 35°, and the angle of elevation to the top of the smokestack is 53°. Find the height, h, of the smokestack exactly. (7 points)

Using Right Δs (Note: if you've learned law of sines, you can approach this another way)



$\tan 35° = \frac{x}{200}$

$200\tan 35° = x$

$\tan 53° = \frac{x+h}{200}$

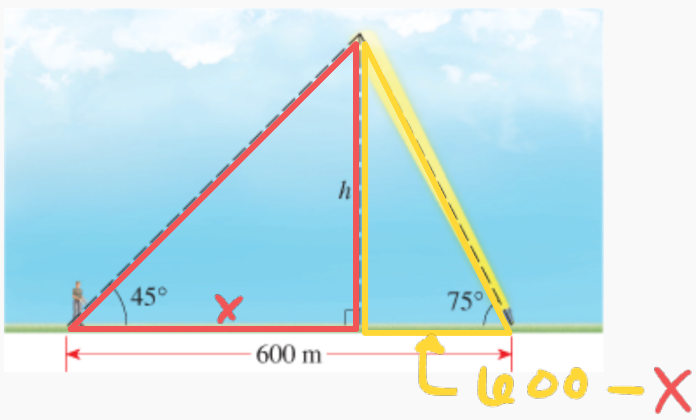
$200\tan 53° = x+h$

$200\tan 53° = 200\tan 35° + h$

$h = 200\tan 53° - 200\tan 35°$

$\approx$

44) **Height of Cloud Cover** To measure the height of the cloud cover at an airport, a worker shines a spotlight upward at an angle 75° from the horizontal. An observer 600 m away measures the angle of elevation to the spot of light to be 45°. Find the height h of the cloud cover.



(7 points)

$\tan 45° = \frac{h}{x}$

$x\tan 45° = h$

$\tan 75° = \frac{h}{600-x}$

$(600-x)\tan 75° = h$

$x\tan 45° = (600-x)\tan 75°$

$x\tan 45° = 600\tan 75° - x\tan 75°$

$x\tan 45° + x\tan 75° = 600\tan 75°$

factor out x

$x(\tan 45° + \tan 75°) = 600\tan 75°$

$x = \frac{600\tan 75°}{\tan 45° + \tan 75°} \Rightarrow h = x\tan 45°$

$h = \frac{600\tan 75°}{\tan 45° + \tan 75°}\tan 45°$

$\approx$